Progressions:

Arithmetic and Geometry progressions

3º E.S.O.

Octavio Pacheco Ortuño

I.E.S. El Palmar

INDEX

Introduction

In this unit students are encouraged to develop their ability to recognise and use numerical structures and patterns in different situations. They should appreciate the value of numerical patterns in making generalisations to explain, simplify or extend their mathematical understanding. They should recognise and work with arithmetic progressions and geometric progressions.

As final task students should make a project about how to model the spread of an epidemic by using progressions.

Course: 3º ESO bilingual Six lessons

Objectives

1. Recognize numerical patterns in progressions.
2. Apply and understand the *n*th term of a progression.
3. Identify arithmetic and geometric progressions.
4. Find the *n*th term of an arithmetic progression; calculate new terms and the sum to *n* terms of an arithmetic progression.
5. Find the *n*th term 'nth' term of a geometric progression and calculate new terms.
6. Represent situations from daily life by using progressions.
7. Generate useful information by using everyday language, mathematical language or a combination of both from modelling with progressions.

Topics

1. Numerical patterns: progressions
2. The *n*th term of a progression.
3. Arithmetic progression: difference.
4. Geometric progression: ratio.
5. Sum to *n* terms of an arithmetic progression.
6. Applications of numerical patterns and progression in daily life.

Timing???

We need six lessons to develop this unit. And an additional lesson for an exam to completed the assessment. We recommended make the exam with another unit, as fractions or proportionality.

The table on the right shows the objectives and topics developed in each lesson.

Lesson	Objetives	Topics
1ª	1,2,3	1 ,2,3
2ª	3,4	2,3,5
3ª	4,5	4,5
4ª	6,7	6
5ª	All	All
6ª	All	All

Activities

Lesson 1

We introduce the concept of numerical progressions and how we can find some patterns. Then we show how to determine a general term, which it could be a formula, so it is called the *n*th term.

We start to introduce the concept of arithmetic progressions and the difference.

We propose the following exercises:

1. Write four more terms for the following progressions:

a) 1, 3, 5, 7, 9, ….. b) 3, 7, 11, 15, …. c) 2, 4, 8, 16, ….
d) 3, 5, 9, 17, …. e) 1, 4, 9, 16, …. f) 1, 2, 4, 7, 11, ….

2. Write the value of a_3 and a_5 for each of the progressions from the previous exercise.

3. Write a_1, a_2 and a_4 of the following progressions:

a) $a_n = 3n$ b) $a_n = 5n - 2$ c) $a_n = n^2 + 3$ d) $a_n = (-3)^n$

4. Find the *n*th term for each progression from exercise 1:

5. Write the first four terms in each of the following arithmetic progressions.

a) $a_1 = 15$; d=8 b) $a_1 = 9$; d=-5
c) $a_1 = -12.8$; d=1.5 d) $a_3 = 32$; d=6

6. Find the *n*th term of the progressions from the previous exercise.

Lesson 2

We continue working with arithmetic progressions and introduce the sum of several terms and how to calculate it.

We propose the following exercises:

1. In each of the following arithmetic progressions, find
(a) the common difference (b) the 10th term (c) the *n*th term

(i) 1, 3, 5, 7, … (ii) 10, 9, 8 , 7, … (iii)1, 2.5, 4, 5.5, ….
(iv) 20, 18, 16, 14, … (v) -25, -20, -15, -10, … (vi)-0,125, -0,25, -0.375, -0.5,…

2. The fifth term of an arithmetic progression is 10 while the 15th term is 40. Write down the first five terms of the arithmetic progression.

3. The fifth and tenth terms of an arithmetic progression are 8 and -7 respectively. Find the 20th and 50th terms of the arithmetic progression.

4. Find the sum of the first eight terms in the following arithmetic progressions:

a) 5, 9, 13, 17,…. b) 32, 29, 26, 23,… c) -25, -20, -15, -10, …

5. Find the sum of all the positive terms of the arithmetic progression 25, 23, 21, 19, ...
Find also the values of *n* for which the sum to *n* terms is 160.

Lesson 3

Now we introduce the second kind of progressions that we are to study; geometric progression and the common ratio, and how to determine the nth term.

We propose the following exercises:

1. Write the first four terms in each of the following geometric progressions.

a) $a_1 = 7$; r=2 b) $a_1 = 5$; r=-4

c) $a_1 = 160$; d=0.5 d) $a_3 = 108$; d=3

2. Find the *n*th term of the progressions from the previous exercise.

3. In each of the following geometric progressions, find

(a) the common ratio (b) the 10th term (c) the *n*th term

(i) 3, 6, 12, 24, 48,… (ii) -10, -40, -160, -640…

(iii) -5, 20, -80, 320, -1280,… (iv) 512, 256, 128, 64, …

4. The second term of a geometric progression is 5 and the fifth term is 625. Find the seventh term.

5. The second term of a geometric progression is 64 and the fifth term is 27. Find the first 6 terms of the geometric progressions.

Lesson 4

We show situations from daily life where we can recognise numerical patterns and use progression to represent those situations.

1. A man who weighs 125 kg is told he must reduce his weight to 80 kg. He goes on a diet and finds he is losing 1.5 kg in weight every week. Find his weight after 11 weeks, and find how long it will take him to get his weight down to 80 kg.

2. A piece of rope with a length of *k* cm is cut into 40 parts such that the length of each part forms an arithmetic progression. Given that the length of the longest part is 324 cm and the difference between two consecutive parts is 8 cm. Find:

a) The length of the shortest part.

b) The value of *k*.

c) The part that has a length of 172 cm.

3. Tom tells two of his friend a secret. The next hour, each of his friends tells the secret to two other friends. On the next hour, their friends each tell the secret to two other friends. How many people are told the secret on eight hours?

4. If the population of Great Britain is 55 million and is decreasing at 2.4% per annum, what will be the population in 5 years time?

5. 100 g of a radioactive substance disintegrates at a rate of 3% per annum. How much of the substance is left after 11 years?

Lesson 5 and 6

To illustrate progressions and developed the use of everyday language to show mathematic result we have use the website:
http://motivate.maths.org/content/MathsHealth/ModellingEpidemics

We have recollected some material and videos from this website to make the following exercises. Also we should watch the following resources from the prior website:

Before Activity 1
Modelling with Mathematics: Introductory video clip (3 min 2 secs)
Before Activity 2
Counter Plague: demonstration Video clip (3 min 47 secs)
After Activity 2
Understanding Counter PlaguePresentation: screenshots from e-Counter Plague to help students interpret the model, and to understand what we might expect to happen, given particular dice settings.

The following next two pages(page 7 and 8) is handout for students.

Modelling with mathematics

Mathematics is often called "the language of the universe". With mathematics, we can describe and make predictions about the behaviour of things around us.

We are going to explore how to model the spread of a epidemic.

Activity 1 :Standing Disease

• **Step 1**: choose one person to be the first case of Standing Disease - the main symptomis being unable to sit or lie down, you just have to keep standing up.

• **Step 2**: it's quite infectious - so that person chooses two more people to be infected.

• **Step 3:** each infected person infects two more people, and so on, until the whole class is infected.

Questions to discuss. Write the answers in your notebook

1. How many steps did it take the disease to infect the whole class?
2. How many people were infected at each stage? How could you describe this number sequence?

3. What difference would it make if 3 people were infected at each stage instead of 2?
4. How could you describe that number sequence?

5. How many steps would it take to infect the whole school?
6. What about Spain? The population is about 45 million
7. How about the UK? The population is about 62 million.

8. How about the whole world? The population is about 7 billion.
9. What about if *n* people were infected at each stage?

How good is this model? Write the answers in your notebook

10. What aspects of the model tell us something useful about how epidemics progress?
11. What does the model predict which doesn't make sense?
12. Can you think of ways to improve it?

Activity 2 Counter Plague

• **Step 0**: put one counter on the table. This is the first infection.
• **Step 1**: throw the die. Use one of the tables below to see how many people are infected, and place the appropriate number of counters to the right of the first counter. The example above uses the LH table. A 5 at Step 0 means that two more people are infected, shown by the two counters at Step 1.
• **Step 2**: repeat for each newly infected person. In the example, a 5 for the top counter means two more infections, while the 2 for the bottom counter means no further infections.
• **Step 3**: repeat until either the epidemic dies out or you run out of counters (meaning the whole population has been infected). In the example, the epidemic died out after the 2 at Step 3.

Table A

Number on die	Number of people infected
1	0
2	0
3	1
4	1
5	2
6	3

Table B

Number on die	Number of people infected
1	0
2	0
3	1
4	1
5	1
6	2

Counter Plague as a model for an epidemic. Write the answers in your notebook
1. Run eight 'epidemics', using the table A for the outcomes of each throw of the die, and record the progress of the epidemic on a bars graph in your notebook.
2. How do the graphs compare?

3. Now run eight 'epidemics' using the table B of outcomes, and record the progress of the epidemic on a graph.
4. How do the graphs compare?

5. How do these compare with the first set of graphs?

How good is the Counter Plague model? Write the answers in your notebook
6. What aspects of the model help us to understand how epidemics progress?
7. Are there important factors which it does not include?
8. Can you think of ways to improve it?

Vocabulary:

Disease-enfermedad
Spread- extender
Die-dado
Die out- desaparecer
Get sick- enfermar
Stage-etapa
Step-paso
Outcomes-resultados
Number sequence-serie numérica

Resources

Tex book: Mathematics3. Basic concepts. Editorial Anaya. J. Colera y otros.

Websites
http://motivate.maths.org/content/MathsHealth/ModellingEpidemics
Download videos, presentation and activities.

http://www.purplemath.com/modules/series.htm
Find more exercises and advanced topics.

Assessment

1. Identify and discover patterns and relationships between the terms of a numerical progression
2. Using the *n*th term of a progression to obtain any term of the progression.
3. Find the *n*th term of an arithmetic progression and the difference.
4. Calculate the sum of n terms of an arithmetic progression.
5. Find the *n*th term of a geometric progression and the ratio.
6. Analyse information and data from daily life situations using progressions.
7. Generate results and conclusions from daily life situations using progressions.

The next table shows the relationship between Objectives and Assessments

Objectives	Assessments
1	1
2	2
3	3 y 4
4	5 y 6
5	7
6	8

The achievement of exit assessments 1 to 5 will be provided by an exam. The rest of assessments will be provided by tasks in lessons 5 and 6.